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Survey of Atmosphere Re-Entry Guidance and Control Methods

RODNEY C. WINGROVE
NASA Ames Research Center, Moffett Field, Calif.

Nomenclature

a = measured acceleration D/W, g's
C_D = drag coefficient
C_L = lift coefficient
D = drag, force along the total velocity vector $(C_DS/2)\rho V^2$, lb
g = local gravitational acceleration, ft/sec^2
h = altitude, distance along radial axis, ft
K = constant feedback control gain
L = lift, force normal to the total velocity vector $(C_LS/2)\rho V^2$, lb
r = distance from planet center, ft
S = surface area upon which force coefficients are based, ft^2
s = differential operator notation
t = time
u = circumferential velocity normal to radius vector, fps
$\bar{u}$ = dimensionless ratio u/(circular orbital velocity), $u/(gr)^{1/2}$
V = total velocity, fps
v = vertical velocity along radial axis, fps
W = weight of vehicle, lb
x = downrange, distance to destination along great circle route, ft
α = vehicle angle of attack
β = atmosphere density decay parameter, ft^{-1}
δ = variation in quantity from that of nominal trajectory
γ = flight-path angle, positive for climb
λ = influence function
Λ = time-varying feedback control gain
ρ = atmosphere density, slugs/ft^3
φ = vehicle roll angle
ψ = heading angle between the instantaneous great circle route and the great circle route to the target
ζ = damping factor
ω_n = natural frequency
$(\ )_f$ = final value
$(\ )_i$ = initial value
$(\ )_{nom}$ = respect to nominal trajectory

Introduction

VARIOUS methods have been investigated for the control of maneuverable vehicles through the atmosphere.[1–53] This paper is a survey of published work in this field. The use of automatic control in entry is treated in Refs. 1–40 and the use of pilot control in Refs. 41–53. These studies have considered closed-loop guidance during atmosphere re-entry for a variety of orbital and superorbital space missions. These investigations have demonstrated many different concepts for keeping the acceleration and heating loads within design limits during entry and for assuring the vehicle's arrival at a desired destination.

This paper first will consider the basic dynamics of entry trajectories and how the dynamics of the trajectory variables are related to the guidance and control problem. The various guidance and control concepts will be described in detail under two general categories: 1) guidance using predicted capabilities, and 2) guidance using a nominal trajectory. Finally, the application of these guidance systems to current vehicle configurations and to entries from circular, supercircular, and abort conditions will be considered.

Dynamics in Entry Motion

Before going into a detailed description of the various guidance methods, it is important to gain some understanding of re-entry trajectory dynamics. These motions are basic to each of the guidance methods, and, as in the analysis of any system, an understanding of the dynamics of the controlled variables is necessary in order to gain insight into the overall control problem.

Dynamics of Constant-Trim Lifting Vehicles

Figure 1 illustrates typical dynamics in atmosphere entry. The motions are shown for a constant-trim (constant L/D) lifting vehicle entering the atmosphere at 300,000 ft where the sensible atmosphere begins and also near local circular satellite velocity ($\bar{u} = 1$). The vehicle is shown to decelerate about an equilibrium glide path. The equilibrium glide path is essentially that one path where the aerodynamic lift force just balances the centrifugal and gravity force along the trajectory.

An understanding of the stability of entry dynamics can be observed in Fig. 1 from the trajectory that is not initially in equilibrium. It can be seen that the dynamics are stable; that is, the motions are oscillatory, and there is a small

Rodney C. Wingrove is a Research Scientist. His early work was in the flight testing of experimental aircraft. The stability and control analysis of handling qualities led Mr. Wingrove to piloted-analog simulation. Later work has been with fixed-base, moving-base, and centrifuge simulations for pilot control during re-entry. Currently he is specializing in the guidance and control of aerospace vehicles. Mr. Wingrove graduated with a B.S. degree in aeronautical engineering from the University of Washington in 1955. He has authored several NACA and NASA publications and several technical papers for professional societies. He is a Member of AIAA.

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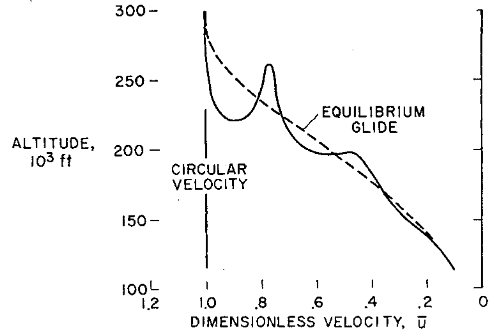


Fig. 1 Dynamics of constant-trim lifting vehicle; $\bar{u}_i$ = 1.0, L/D = 0.5, W/C_DS = 50 psf.

amount of damping. Many studies (i.e., Refs. 34, 37, 38, 54–59) have analyzed these dynamics. Reference 38 has shown that the frequency and damping of the oscillations, at local points along the trajectory, can be approximated by the following:

Damping

$$2\zeta\omega_n \approx \frac{1}{\bar{u}}, \left(\frac{\text{rad}}{\text{unit of } \bar{u}}\right) \tag{1}$$

Frequency

$$\omega_n^2 \approx \beta r \frac{1 - \bar{u}^2}{(D/W)^2}, \left(\frac{\text{rad}}{\text{unit of } \bar{u}}\right)^2 \tag{2}$$

These motions are written with respect to dimensionless velocity $\bar{u}$. The equation for the frequency contains the term $(1 - \bar{u}^2)$; hence, the trajectory motions possess static stability below circular velocity ($\bar{u} < 1$) and are statically unstable above circular velocity ($\bar{u} > 1$). The damping, which was observed to damp the motion lightly in Fig. 1, is proportional to $1/\bar{u}$, so that the damping increases as the velocity decreases.

The short-period motions of re-entry vehicles have been analyzed in many studies (i.e., Refs. 57–62). The work in Ref. 59 has illustrated that for practical entry configurations the short-period oscillations do not significantly couple with the long-period trajectory dynamics. Most preliminary guidance studies, then, such as those discussed in the following sections, consider primarily point-mass equations in the analysis of the long-period motions.

Relationship of Control and Dynamics

Figure 2 shows the time history of the deviation of the four state variables (in the plane of the trajectory) from a nominal equilibrium path for a typical controlled trajectory. The important feature in the dynamics is apparent from the difference in time at which the curves reach a maximum.

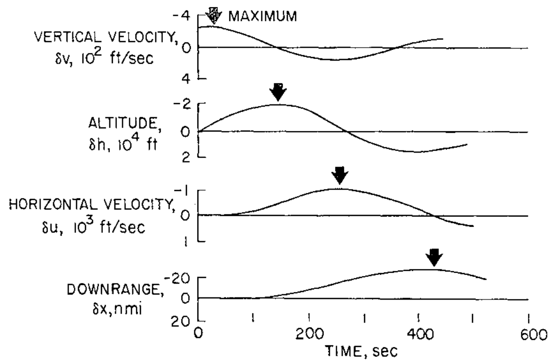


Fig. 2 Time history of deviation in state variables from nominal equilibrium path; $\bar{u}_i$ = 1.0, h_i = 300,000 ft, γ_i = −1.0°, $(L/D)_{nom}$ = 0.25.

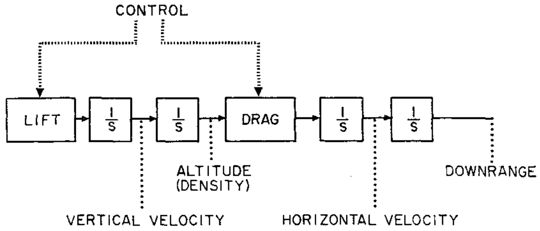


Fig. 3 Relationship of control and dynamics.

The deviations in the range are anticipated by deviations in horizontal velocity; deviations in horizontal velocity are anticipated by deviations in altitude; deviations in altitude are anticipated by deviations in vertical velocity. These characteristics can be explained with the aid of the block diagram in Fig. 3. This diagram, which is a simplified representation of the equations of motion, illustrates the relationship of the control forces to the trajectory variables.

The lift force is essentially in the vertical direction. The vertical force affects the rate of change of vertical velocity. The integration $(1/s)$ of the vertical velocity gives the variation in altitude (or, what is more important, variation in density). This change in density affects the drag force and thus affects the rate of change of horizontal velocity. An integration of the horizontal velocity gives variations in the range along the path.

To control range, the rate at which horizontal velocity is changing must be controlled; thus, the drag must be controlled. Drag can be regulated principally by either changes in the configuration (i.e., trim changes or drag brake) or, what is more important, changes in density. For instance, if at any time in the trajectory the range must be extended, the lift force* is increased in the vertical direction to raise the vehicle into less dense atmosphere, thus reducing the rate of change of horizontal velocity and extending the range.

Important features in re-entry control from Fig. 3 will be referred to in the following discussions. The control of range by lift constitutes a fourth-order system (the product of four integrations, $1/s$). It can be reasoned that this fourth-order system, like any other classical fourth-order system, needs four feedback quantities to shape the desired response. Certain measurements that may be used for control feedback are listed in Table 1.

As shown in the table, the first-order feedback quantity can be the measurement of the rate of change in altitude (vertical velocity) or can be one of those measurements that reflect the corresponding rate of change in density (i.e., air-pressure rate, acceleration rate, temperature rate). For the second-order feedback (altitude), those measurements that reflect air density also may be used and, in fact, are desirable because it is the actual density, not the altitude itself, that affects the aerodynamic forces. The third-order

Table 1 Relationship of measurements to entry dynamics

Measuring devices	$\frac{1}{s}$ 1st order feedback	$\frac{1}{s}$ 2d order feedback	$\frac{1}{s}$ 3d order feedback	$\frac{1}{s}$ 4th order feedback
Inertial unit or tracking	Vertical velocity	Altitude	Horizontal velocity	Downrange
Accelerometer	Drag acceleration rate	Drag acceleration	Integration of drag acceleration	Second integration of drag acceleration
Temperature sensor	Air or skin temperature rate	Air or skin temperature	---	---
Pressure sensor	Air pressure rate	Air pressure	---	---

* Reference 34 contains a more complete description of control in which lift and drag forces are highly coupled.

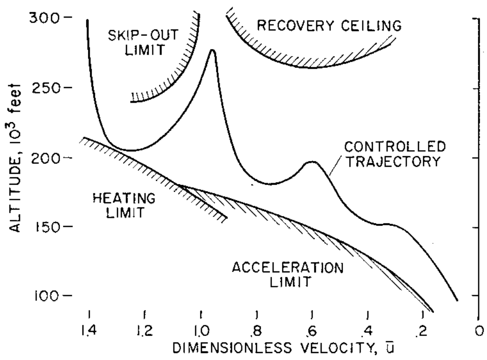


Fig. 4 Typical operating boundaries.

feedback quantity can be some measurement of horizontal velocity, and the fourth-order feedback quantity is a measurement of the range along the trajectory. It should be noted that the measurements of acceleration[22, 23] and temperature[25, 50] also can be used to keep the vehicle within design load limits as well as to provide control feedback.

Control Boundaries

The re-entry guidance system must, in addition to controlling range, prevent the vehicle from exceeding certain constraints. The operational constraints that must be avoided during re-entry are illustrated in Fig. 4. If the vehicle is at supercircular velocity ($\bar{u} > 1$) and concurrently reaches too high an altitude, even though holding full negative lift, it will skip uncontrolled out of the atmosphere. It should be noted that the skip-out boundary is a three-dimensional boundary and is defined by altitude, horizontal velocity, and vertical velocity. A typical boundary for conditions near zero vertical velocity is illustrated on the altitude-horizontal velocity plot in Fig. 4. Also, if the recovery ceiling is exceeded in flight below circular speed, the vehicle is flying too slow to sustain altitude and is unable to check its descent before passing through the lower boundary. Below the lower boundaries the vehicle is flying too fast to maintain deceleration and/or heating rates at tolerably low values. One control limit not illustrated is time. If a vehicle using an ablator or heat sink rather than radiative cooling spends too much time in the atmosphere, then the heat generated over this long time period may exceed the design limit.

The following discussion will describe various guidance methods that can be used to regulate the aerodynamic forces on the vehicle so that these operating boundaries are not exceeded and the vehicle reaches a desired destination.

Discussion of Guidance Methods

This section will outline in detail the various guidance methods. The literature has indicated that each of these methods can yield satisfactory trajectory control. The differences in these systems which will be pointed out are the relative advantages or disadvantages with regard to the ability to handle off-design entrance conditions, the on-board computer requirements, the flexibility to maintain trajectories with minimum heating or minimum acceleration, and the information that the guidance equations give a pilot.

The guidance methods are presented under two general classifications: guidance using predicted capabilities and guidance using a nominal trajectory. These are broken further into subgroups. This does not necessarily imply that all guidance systems fall under only one of these categories.

Guidance Using Predicted Capabilities

Figure 5 illustrates guidance using predicted capabilities. There is the choice of a number of paths within the typical

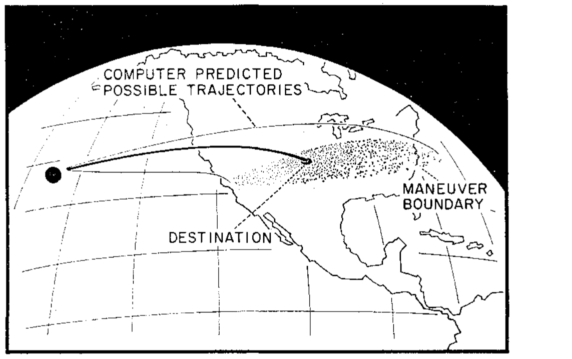


Fig. 5 Guidance using predicted capabilities.

maneuver boundary, as shown in the figure. The guidance system predicts the path by which the vehicle will reach the desired destination without violating the heating and acceleration limits. This general group contains those methods that predict possible future trajectories and do not use a stored nominal trajectory. The discussion of this group will consider prediction by either "fast-time" solution or approximate "closed-form" solution of the equations of motion.

Fast-time prediction

The use of fast-time prediction in conjunction with automatic control has been studied in Refs. 20 and 32–34 and with pilot control in Refs. 46 and 51.† The basic concept of these systems is that the differential equations of motion are solved by a "fast" computation in the airborne computer to determine possible future trajectories. Repetitive solutions are made from the continuously measured state variables along the trajectory.

Typical information needed to make a prediction of the path in the plane of the trajectory is the measurement of four state variables (e.g., v, h, u, x) and the estimation of two vehicle parameters (e.g., L/D, W/C_DS). There are various combinations of these quantities which can be used to predict the path. For instance, in the work of Refs. 32, 33, and 46, the altitude and W/C_DS quantities are replaced by the drag acceleration measurement.

The desired trajectory can be found by iteration procedures such as in the automatic control studies of Refs. 20 and 32–34. Typical controlled trajectories from Ref. 33 are shown in Fig. 6. These trajectories are for a vehicle entering the atmosphere at supercircular velocity with the desired destination 8000 miles from the entrance point. The figure illustrates control where the re-entry conditions used in the prediction are in error from the actual re-entry conditions. A controlled trajectory is shown for a change

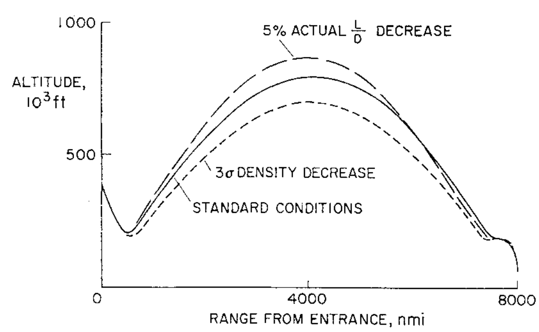


Fig. 6 Trajectories from Ref. 33 controlled by fast-time prediction; $\bar{u}_i$ = 1.4, range-to-go = 8000 naut miles, $-0.5 < L/D < +0.5$.

† Additional studies also have been made by the Bell Aerosystems Company and by Bendix Systems Division.

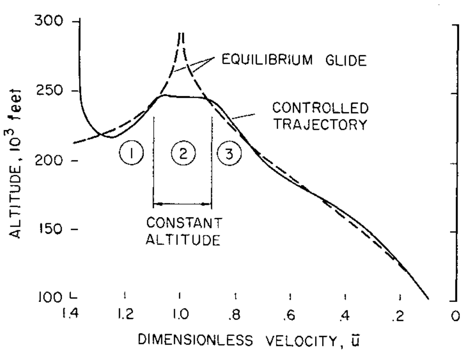


Fig. 7 Closed-form prediction method from Ref. 29.

in the actual trim L/D from the estimated L/D used in the prediction. Also, a change in the actual density from the predictor standard-density profile is illustrated. Without knowing that these errors exist, the system is shown to guide the vehicle to the destination.

In pilot control, the fast-time prediction can be used to display the vehicle maneuver capability with respect to possible destinations.[46, 51] The solution of the trajectories also can give future heating, acceleration, and altitude excursions that the pilot may use to anticipate problems along the trajectory.

The main advantage of the fast prediction method is that it affords the ability to handle any possible flight condition. This ability to predict range, acceleration, heating, etc., makes it a flexible control method. The principal disadvantage of the system is the stringent on-board computer requirements for the fast-time computation. Studies to date indicate that the repetitive prediction must be made every 2 to 5 sec for those entry trajectories where conditions are changing rapidly. In smooth gliding trajectories, computation times on the order of tens of seconds may be permissible.

Closed-form prediction

Closed-form prediction systems differ from fast-time prediction systems in that, instead of integrating the equations of motion, they employ an approximate explicit solution in which all or part of the possible trajectories are considered.

Numerous studies have been made to develop analytical descriptions of entry trajectories (e.g., Refs. 64–80). For those parts of the trajectories out of the influence of the atmosphere, closed-form solutions of Newton's two-body drag-free equations can be used. Some common closed-form solutions that can be used for prediction within the atmosphere are the following.

A vehicle controlled to constant value of drag acceleration D/W has the following closed-form approximation of range from the initial velocity $\bar{u}_i$ to the final velocity $\bar{u}_f$:

$$\frac{x}{r} \approx \frac{\bar{u}_i^2 - \bar{u}_f^2}{2(D/W)} \tag{3}$$

For a vehicle flown at constant altitude (constant density), the acceleration along the resulting trajectory‡ is

$$\frac{D}{W} = \left(\frac{D}{W}\right)_i \left(\frac{\bar{u}}{\bar{u}_i}\right)^2$$

and the predicted range is

$$\frac{x}{r} = \frac{\bar{u}_i^2}{(D/W)_i} \ln\left(\frac{\bar{u}_i}{\bar{u}_f}\right) \tag{4}$$

‡ This equation is valid for the case where lift in the vertical plane is modulated by changing the roll angle of the vehicle. Reference 77 contains the equations where the lift and drag both are varied by changes in the vehicle pitch angle.

For a vehicle controlled to a constant flight-path angle γ, the resulting acceleration along the trajectory is approximately

$$\frac{D}{W} \approx \left(\frac{D}{W}\right)_i \left(\frac{\bar{u}}{u_i}\right)^2 + \beta r \bar{u}^2 \sin\gamma \ln\left(\frac{\bar{u}}{\bar{u}_i}\right)$$

and the predicted range is

$$\frac{x}{r} \approx \frac{\cot\gamma}{\beta r} \ln\left(\frac{D_f}{D_i}\frac{\bar{u}_i^2}{\bar{u}_f^2}\right) \tag{5}$$

This constant flight-path angle solution also approximates the trajectory of a nonlifting vehicle along a steep flight path where the gravity and centrifugal forces are negligible compared to the radial drag force. The addition of lift force normal to the path will change the flight-path angle along the trajectory in the following manner:

$$\sin\gamma \approx \sin\gamma_i - (L/D)\ln(\bar{u}/\bar{u}_i)$$

and the acceleration is modified by

$$\frac{D}{W} \approx \left(\frac{D}{W}\right)_i \left(\frac{\bar{u}}{\bar{u}_i}\right)^2 + \beta r \bar{u}^2 \sin\gamma_i \ln\left(\frac{\bar{u}}{\bar{u}_i}\right) - \frac{\beta r \bar{u}^2}{2}\frac{L}{D}\ln^2\left(\frac{\bar{u}}{\bar{u}_i}\right)$$

This closed-form solution is for small flight-path angles and is valid where the gravity and centrifugal forces are small, such as near-circular velocity, or where the aerodynamic forces predominate, such as near the bottom of a skip maneuver.

For a smooth gliding constant L/D trajectory, the approximation for an equilibrium glide can be used. A balance of the lift force with the gravity and centrifugal forces along the trajectory results in

$$\frac{D}{W} \approx \frac{1 - \bar{u}^2}{L/D}$$

and the predicted range is

$$\frac{x}{r} \approx \frac{1}{2}\frac{L}{D}\ln\frac{1 - \bar{u}_f^2}{1 - \bar{u}_i^2} \tag{6}$$

Equation (6) is good for either the supercircular or the subcircular velocity portions of the trajectory; however, at circular velocity the range prediction becomes infinite. Generally, the required prediction is made by considering constant altitude, or constant acceleration, or ballistic paths from supercircular to subcircular velocities.[72, 75, 81–83]

Figure 7 illustrates a typical closed-form prediction system considered for control from supercircular velocity. In this guidance method,[29] the trajectory is divided into three phases of control, with each phase amenable to a closed-form solution. Along the supercircular portion of the glide path, a prediction is made indicating the range achievable from the measured conditions for a constant altitude path followed by equilibrium glide to the desired destination. The vehicle flies up the supercircular path until the predicted range corresponds to the range to go. At that time the vehicle is controlled onto the constant altitude path. Prediction is continued while controlling along the constant altitude path and following along the equilibrium path.

Guidance by means of simple closed-form solutions has an advantage in the modest requirement for on-board computation and storage. But, guidance by means of simple closed-form solutions usually does not include the flexibility to handle markedly off-design conditions. This is because most closed-form solutions do not take account of all four state variables in the solution of possible trajectories. With Eq. (6), for example, the prediction of equilibrium glide considers only two state variables. To handle off-design conditions, additional terms, often with much complexity, must be added to the simple closed-form solutions, thus increasing the computer requirements.

This method has the disadvantage that only those design trajectories which can be described analytically can be used. There is not the flexibility of using any desired trajectory profile, such as was the case for the fast-time prediction.

Guidance Using Nominal Trajectory

Guidance about a nominal trajectory is illustrated in Fig. 8. In this method, the state variables along the nominal path are precomputed and stored on-board. The variations in the measured variables from the stored values are used in guidance either to control onto the nominal trajectory (path controller) or to establish a new trajectory to reach the destination (terminal controller).

For this guidance scheme, a nominal trajectory that is the most desirable flight path must be selected. For guidance from subcircular velocities, a constant L/D trajectory close to the center of the vehicle-maneuvering capability is generally selected. In many cases, particularly above circular velocity, a varying L/D trajectory is desired because of heating loads, guidance sensitivity, and range capability. The desired nominal trajectory can be selected prior to entry by optimization procedures. References 39 and 84–88 illustrate the application of various optimization procedures to the selection of re-entry trajectories.

The nominal trajectory guidance method stores the state variables, and in certain cases the feedback control gains, as functions of a given independent variable along the path. The independent variable may be the obvious quantity time, or it may consist of one state variable or some combination of several. In the literature surveyed, the independent variables most frequently used were the time (t) and velocity (u), and they both are demonstrated in this paper. Those studies that used velocity as the independent variable generally indicate more capability to control about a given nominal trajectory than those studies that used time. For instance, Ref. 40 illustrates that, with velocity as the independent variable, only one nominal trajectory is required for normal as well as for off-design re-entry conditions.

Path control using fixed feedback gains

Many studies[4, 8–10, 14, 15, 22, 27, 28, 38] have considered the use of constant feedback gains for guidance to the nominal trajectory. The guidance law for a system with constant feedback gains K and time as the independent variable is

$$\text{control } L/D = (L/D)_{\text{nom}} + K_1\delta v + K_2\delta h + K_3\delta u + K_4\delta x|_t \quad (7)$$

where $(L/D)_{\text{nom}}$ is the value along the nominal trajectory, and the δ quantities represent the variations of the measured state variables from the stored values along the trajectory.

The guidance law for constant feedback gain control when velocity is used as the independent variable is

$$\text{control } L/D = (L/D)_{\text{nom}} + K_1\delta v + K_2\delta h + K_4\delta x|_u \quad (8)$$

In those applications of these guidance laws where the controlled trajectory does not deviate greatly from the nominal, one or more of the feedback terms may possibly be unused. This is because, as shown in Fig. 1, the basic dynamics in entry do possess some stability. The effect of the various terms in the guidance law of Eq. (8) on the trajectory dynamics is illustrated in Fig. 9. Figure 9a presents the range error δx along the trajectory for control from circular velocity. With all terms included in the control, it can be seen that the initial range error is driven to zero and the overshoot is small; hence, the system response is good. The control can be maintained with the $K_1\delta v$ or $K_2\delta h$ terms removed, but the response is oscillatory and not so well damped as when all of the terms in the control equation are used.

All four state variables should be used in applying Eq. (8) in conditions where the path deviates greatly from the nominal or in those portions of the trajectory where the guidance situation is extremely sensitive to small changes in control. This is illustrated in Fig. 9b for control from supercircular velocity. With all of the terms in the guidance law the range error is driven to zero, but removing either the $K_1\delta v$ or $K_2\delta h$ term causes system instability.

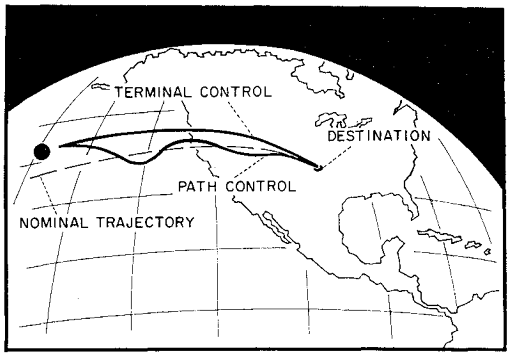
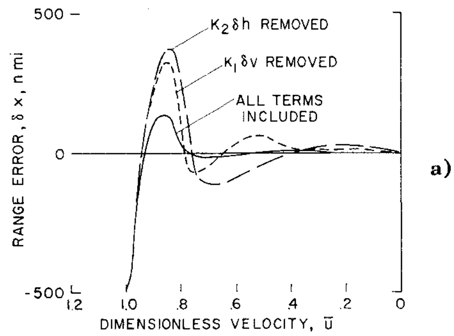


Fig. 8 Guidance methods using a nominal trajectory.

Some understanding of guidance about a nominal trajectory and the stability of this type of system is derived from the fact that Eq. (8) represents a third-order control system with respect to velocity. As in other third-order systems, the outer loop gain K_4 could possiby be high enough to produce system instability. From Ref. 38, using Routh's criterion for stability, K_4 must be positive, and also

$$K_4 < \frac{[(1 - \bar{u}^2) - K_2\beta(D/W)^3][(1/\bar{u}) - K_1(gr)^{1/2}]}{r\bar{u}} \quad (9)$$

Equation (9) permits qualitative determination of the upper limit of K_4 and permits, also, observation of the influence of K_1, K_2, K_4, and $\bar{u}$ on trajectory stability. The addition of K_2 allows stable response above circular velocity. Increasing the magnitude of K_1 and K_2 permits increase of the upper limit of K_4. It is important to observe that the upper limit of K_4 will increase as $\bar{u}$ decreases. To maximize range capability and to drive the range error to zero, it is



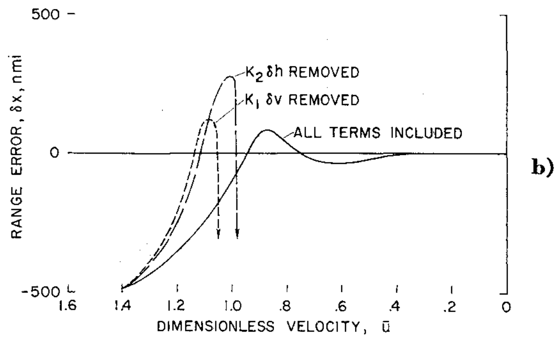


Fig. 9 Effect of control terms in fixed-gain method of Eq. (8); initial range error = −500 naut miles, K_1 = −1 × 10^{-3}/fps, K_2 = −3 × 10^{-5}/ft, K_4 = 1 × 10^{-6}/ft, −0.5 < L/D < +0.5. a) Entry at circular velocity; $\bar{u}_i$ = 1.0. b) Entry at supercircular velocity; $\bar{u}_i$ = 1.4.

desirable to use the maximum possible value of range gain which will still permit some margin of system stability. Stability considerations indicate that the value of K_4 used must be low at high velocity and can be increased only for lower velocities.

Numerous system concepts have proposed simplified versions of constant feedback control methods. For instance, electrical compensation networks (i.e., lead or lag) can replace the number of measurements of state variables. From the relationships, $(K_1\delta v + K_2\delta h)$ is equivalent to $(K_1 + K_2/s)\delta v$, and $(K_3\delta \bar{u} + K_4\delta x)$ is equivalent to $(K_3 s + K_4)\delta x$, and it is seen that the number of stored variables can be reduced. Carrying this to the extreme, a simple guidance law takes the form of

$$\text{control}\,\frac{L}{D} = \left(\frac{L}{D}\right)_{\text{nom}} + \left(K_1 s + K_2 + \frac{K_3}{s} + \frac{K_4}{s^2}\right)\delta a\Big|_t \quad (10)$$

where a is the measured drag acceleration. This system uses only a program of an acceleration time history.

The fixed-gain systems discussed will work in many re-entry applications, but varying gains, which will be discussed next, allow more freedom to shape the desired response along the trajectory and usually improve system performance.

Terminal control using influence coefficients

One effective method of control about a nominal trajectory is to employ influence coefficients. These functions can be solved from the set of differential equations adjoint to the linearized perturbations of the equations of motion about a nominal path. The method of adjoint functions is discussed in Ref. 89, and applications have been made to re-entry guidance in Refs. 5, 13, 19, 23, 24, 26, and 40.

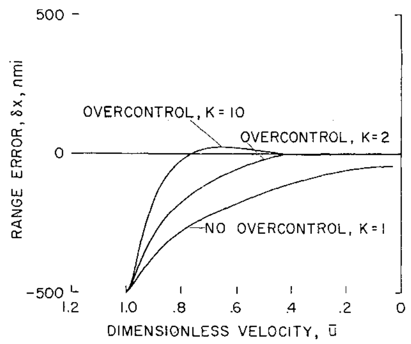


Fig. 10 Effect of overcontrol in terminal controller method; $\bar{u}_i = 1.0$, initial range error = −500 naut miles, $(L/D)_{\text{nom}} = 0.25$, $-0.5 < L/D < +0.5$.

The use of influence coefficients in control will be illustrated for guidance about subcircular equilibrium glide trajectories. From Eq. (6), it can be seen that the control L/D for a given range to go can be approximated by

$$\text{control}\,\frac{L}{D} = \frac{x}{\frac{1}{2}r\ln[(1 - \bar{u}_f{}^2)/(1 - u_i{}^2)]} \quad (11)$$

Rewritten in terms of linear prediction away from a nominal trajectory, this equation becomes

$$\text{control}\,\frac{L}{D} = \left(\frac{L}{D}\right)_{\text{nom}} + \frac{x - \frac{1}{2}r(L/D)_{\text{nom}}\ln[(1 - \bar{u}_f{}^2)/(1 - \bar{u}_i{}^2)]}{\frac{1}{2}r\ln[(1 - \bar{u}_f{}^2)/(1 - \bar{u}_i{}^2)]}$$

$$= \left(\frac{L}{D}\right)_{\text{nom}} + \frac{\partial(L/D)}{\partial x}\,\delta x \quad (12)$$

This simplified control L/D primarily depends upon two state variables, u and x. To take account of deviations in the other state variables from the nominal trajectory, influence coefficients can be used in the following manner:

$$\text{control}\,\frac{L}{D} = \left(\frac{L}{D}\right)_{\text{nom}} + \frac{\partial(L/D)}{\partial x}\,(-\lambda_1\delta v - \lambda_2\delta h + \delta x)\Big|_u \quad (13)$$

In this equation, λ represents influence coefficients that relate changes in the final range x to variations in v and $\dot{h}$ along the trajectory.

For the special case, using subcircular equilibrium glide paths as the nominal trajectories,[19] the influence coefficients can be approximated in closed form as a function of velocity:

$$\lambda_1 \approx \left(\frac{r}{g}\right)^{1/2}\frac{\bar{u}}{1 - \bar{u}^2}\ \text{naut miles/fps} \quad (14)$$

$$\lambda_2 \approx \frac{2 \times 10^{-4}}{(L/D)_{\text{nom}}(1 - \bar{u}^2)}\ \text{naut miles/ft} \quad (15)$$

This guidance law, as stated, is good only for the linear region near the nominal trajectory. To handle conditions far removed from the nominal, overcontrol is needed to assure that the destination remains within the vehicle maneuver capability. Also, overcontrol is needed because the prediction may be in error with measurement errors, atmosphere density variations, aerodynamic trim variation, and other uncertainties. The overcontrol can be achieved by the addition of the factor K in the control equation:

$$\text{control}\left(\frac{L}{D}\right) = \left(\frac{L}{D}\right)_{\text{nom}} + K\,\frac{\partial(L/D)}{\partial x}\,(-\lambda_1\delta v - \lambda_2\delta h + \delta x)\Big|_u \quad (16)$$

The effect of the overcontrol ($K > 1$) with this terminal controller method is illustrated in Fig. 10. For the particular entrance condition shown, without overcontrol ($K = 1$) the destination is not reached; with $K = 2$ and $K = 10$, the destination is reached. The control of these trajectories, which have entrance conditions identical to those of Fig. 9a, illustrates the response of the terminal controller (Fig. 10) as compared with the response of the fixed feedback gain control (Fig. 9a).

Overcontrol also can be provided by a bang-bang terminal controller. Bang-bang terminal controllers use maximum control input until the final range error is driven to zero, that is,

$$\delta x|_{t_f} = 0 = -\lambda_1\delta v - \lambda_2\delta h - \lambda_3\delta u + \delta x|_t \quad (17)$$

where λ represents time-varying influence coefficients that relate the final response at t_f to the conditions at t. In practice, some deadband is used to prevent a continuous limit-cycle operation. When final value prediction falls within this deadband, then only nominal control effort is required to reach the destination. Guidance methods of this type have been used to control both lift-modulated[13] and drag-modulated[21] vehicles from circular velocity and also to control a combination lift- and drag-varying configuration in pull-out maneuvers at supercircular velocities.[26]

For guidance using influence coefficients, any value of overcontrol between $K = 1$ (essentially constant trim) and $K = \infty$ (bang-bang) can conceivably be used to obtain the needed overcontrol. In determining the best value of overall gains to use in the control system of this general type, the optimization procedures as described in the next section can be applied.

Optimized feedback gains

In optimizing the set of time-varying feedback control gains, there is the choice of a performance index that the particular optimization procedure will attempt to minimize. Although what may be optimum in one case may not be optimum in another, the important consideration is that the system satisfies the requirements and constraints established. The optimization procedures provide the technique for carrying out this control system analysis in a direct and systematic manner. This paper will outline those optimization studies which have dealt directly with re-entry guidance and will not attempt to discuss all the optimization procedures that have been developed.

In Ref. 23, the lambda matrix control scheme is applied to the guidance of a low L/D vehicle entering the atmosphere at supercircular velocity. Lambda matrix control is a terminal controller that minimizes the mean-square control deviation. The control law for re-entry guidance is of the form

$$\text{control}\left(\frac{L}{D}\right) = \left(\frac{L}{D}\right)_{\text{nom}} + \Lambda_1\delta v + \Lambda_2\delta h + \Lambda_3\delta u + \Lambda_4\delta x|_t \quad (18)$$

where Λ represents the time-varying gains determined by the optimization procedure. The time-varying gains are precomputed and depend upon the design nominal path.

In Ref. 39, the neighboring optimum control scheme is used in the guidance of a low L/D vehicle from supercircular velocity. Neighboring optimum control uses time-varying gains that optimize one of the terminal quantities while satisfying other terminal constraints.

In Ref. 34, the parametric expansion method is applied to the control of a medium L/D vehicle re-entering at circular velocity. In this study, the performance index is based on the integral-squared error of the variations in dynamic pressure, range-to-go, vertical velocity, temperature, and the angle of attack. Additional considerations for the crossrange requirements are the integral-squared error of the crossrange-to-go and bank angle. Weighting functions are used with the various terms to determine the time-varying gains.

These optimization studies for re-entry control have all used time as the independent variable. Consideration has not yet been given to the use of state variables (i.e., velocity, range-to-go, etc.) as the independent variable to enable these optimized systems to cope with conditions markedly off the nominal.

To summarize the discussion of guidance methods using a nominal trajectory, Fig. 11 illustrates the various schemes. The constant feedback method of control uses fixed values of feedback gains to control onto the nominal trajectory, as shown in the figure. In the constant-trim terminal control method, influence coefficients are precomputed by linear analysis of the trajectory equations. From these influence coefficients, a vehicle trim position is determined which will result in a flight trajectory that will terminate precisely at the desired destination. The bang-bang terminal control method uses maximum control input until the nominal control can be employed to establish a new trajectory to the destination. The optimized feedback gain method uses time-varying feedback gains that have been optimized either as a terminal controller or as a path controller about the nominal trajectory. The fixed feedback gain system has been found in the literature to provide good control primarily below circular velocity. The other varying feedback gain systems (terminal control and optimized feedback gains) generally provide better overall capability.

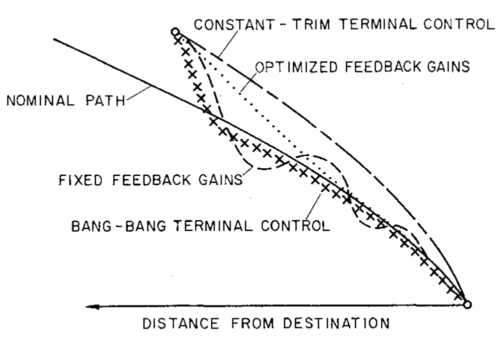


Fig. 11 Various guidance methods using a nominal trajectory.

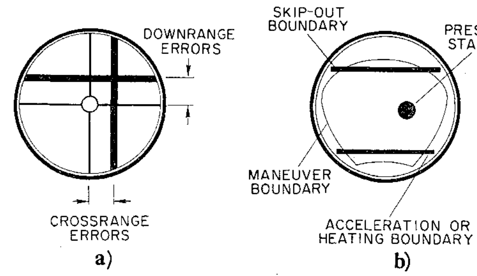


Fig. 12 Typical guidance displays: a) nominal trajectory, b) predicted capabilities.

The use of the nominal trajectory guidance method provides a simple on-board guidance computation. The use of many stored trajectories and stored feedback gains implies a large on-board storage requirement. Many studies have found, though, that, with the proper selection of control variables, a single reference trajectory might be adequate for most entry missions, thus requiring only a modest storage requirement.

Pilot Participation in Guidance

The information that the pilot receives from the guidance computer comes from displays of the type shown in Fig. 12. Such information is shown to depend upon the particular guidance equations. As seen in Fig. 12a, the display for the nominal trajectory method gives only information with respect to the nominal trajectory. This type of information, which can be presented on command pointers similar to those in current aircraft, does not give the overall situation. For the guidance method that uses predicted capabilities, however, additional information is available, as shown in Fig. 12b. Future altitude, acceleration, or heating problems, as well as the information regarding the availability of destinations within the maneuver boundary, can be presented to the pilot for decisions on the overall situation.

In addition to the information coming from the guidance equations, the pilot can monitor acceleration sensors, temperature sensors, radio signals, and other data essential in entry guidance. When necessary, the pilot can override incorrect or questionable signals, provide the control responses, and make decisions on the overall mission profile.[41–53]

Guidance System Applications

Preceding sections of this paper presented the details of various laws. This section examines these laws for some typical vehicle applications.

Typical Vehicle Control Systems

The regulation of the lift and drag forces has been considered in the foregoing general discussions. In vehicle control, the lift or drag can be regulated by the angle of attack (α) and roll angle (φ), as illustrated in the following two control systems.

For the control of the Dyna-Soar boost glider for subcircular velocities, it has been proposed[15, 90] to control downrange in the following manner:

$$\text{control } \alpha \Big]_{\alpha_{\min}}^{\alpha_{\max}} = \alpha_{\text{nom}} + K_1 v + K_4 \delta V|_x \quad (19)$$

The nominal trajectory consists of a stored program of total velocity vs range, and a filtered vertical velocity term is used for damping. Crossrange is controlled by

$$\text{control } \varphi = K\psi \tag{20}$$

where ψ is the angle between the instantaneous heading of the velocity vector and the heading of the great circle line between the present position and the landing site.

For illustrative purposes, the same general method of control can be considered about a nominal trajectory for a low L/D, constant-trim lifting body which is typical of those considered for entry at supercircular velocities. For a constant-trim lifting body, the roll angle can be used to control the lift vector in the vertical plane in the following manner[28]:

$$\text{control } |\varphi| \Big]_{180°}^{0°} = \varphi_{\text{nom}} + K_1\delta v + K_2\delta a + K_4\delta x|_u \tag{21}$$

The three state variables of the nominal trajectory are stored as a function of the velocity. The gains K can conceivably be constant, but, for the best control system, they should be variables determined by optimization design procedures. This command equation determines the magnitude of the roll angle. The sign of the roll angle is determined by the bearing to the landing site.

Guidance System Integration

A block diagram of a typical re-entry guidance and control system is illustrated in Fig. 13. The primary control loop is through the inertial platform sensors and on-board computer. The sensor information is continuously fed into the computer to estimate the vehicle velocity and position with respect to the planet, and this information is used in the computation of the guidance logic. The output of the guidance computer is fed into an autopilot or displayed to a human pilot. The pilot or autopilot thus controls the vehicle in response to this guidance logic input. The guidance computation time to maintain system stability must be on the order of 2 to 5 sec for those entry trajectories where conditions are changing rapidly. In smooth gliding trajectories, system stability can be maintained with computation times on the order of tens of seconds.

The measurements obtained from inertial navigation units incorporate and compound errors resultant from errors in initial alignment of the platform, errors in initial position and velocity information, and errors in the measuring equipment. The inertial navigation unit has a basic instability in the vertical coordinate[63] such that air data or tracking information may be desired to either update or damp the vertical position error.

Updating in the mission can come from tracking data when they are available. This could be vehicle tracking from the ground or planet and star tracking from the vehicle. It should be noted that the tracking information will be affected adversely by a plasma sheath encompassing the vehicle during a portion of the entry (approximately 150,000- to 300,000-ft alt) and that for the case of ground tracking a large number of tracking stations may be required in order to cover the possible re-entry paths on the planet.

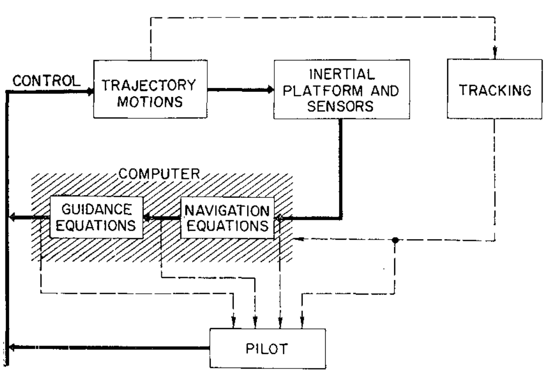


Fig. 13 Typical guidance and control system for re-entry.

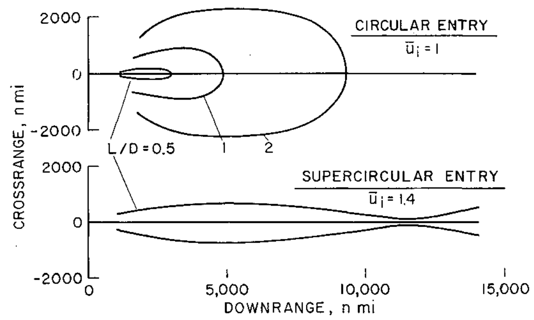


Fig. 14 Attainable ground area.

Guidance Capabilities in Entry

Attainable Ground Area

The previous discussions have outlined possible guidance methods. The attainable ground area demonstrated in the studies of these systems to date is presented in Fig. 14. As shown for re-entry from circular orbit, the available lift-to-drag ratio of the vehicle is extremely important in determining the attainable ground area. For those vehicles with an L/D of 0.5, such as the proposed lifting capsule configurations, there are about ± 200 miles of available crossrange and 2000 miles of available downrange. For the higher L/D vehicles, such as the Dyna Soar with an L/D between 1 and 2, there is appreciable ground area available for maneuvering from orbit.

Most guidance studies of closed-loop control for entry at supercircular velocity have considered a low L/D vehicle. As shown in Fig. 14, a vehicle with an $L/D = 0.5$ at supercircular velocity has about 600 miles of maximum crossrange capability, and it is interesting to note that at about 12,000 miles from entry the crossrange is limited. This is because the vehicle is one-half the way around the earth, and the great circle routes from the initial entry converge at this point.

The entry at supercircular velocity, such as encountered in the return from a lunar mission, presents particular guidance problems, which will be discussed next.

Guidance from Supercircular Velocities

To make a successful entry from supercircular velocities, the vehicle must be on a trajectory within a safe entry corridor.[91–93] This corridor is shown in Fig. 15 in terms of the overshoot boundary, where the vehicle will just stay within the atmosphere, and the undershoot boundary, where the vehicle will reach a specified acceleration or heating limit. From within this corridor, the re-entry guidance system can control the vehicle either through the atmosphere or on a skipping maneuver out of the atmosphere (as shown in Fig. 6) to the destination. The choice of the particular design trajectory is a compromise between acceleration and heating loads, guidance sensitivity, and the range-to-go to the target.

The available range control within the operational corridor for an $L/D = 0.5$ vehicle is shown in Fig. 16. The upper boundary of the corridor is determined by the shallowest entrance at which the vehicle can remain within the atmosphere while holding full negative lift. Control near this limit is sensitive because almost full negative lift is needed to keep the vehicle within the relatively low-density portion of the atmosphere, and little control is available for the control of range. Also, near the overshoot boundary the vehicle spends a long time within the atmosphere, so that the total heat load limit can be exceeded for the longer ranges.

The undershoot limit is determined by the acceleration limit of the vehicle or crew. The value of 10 g, which usually is used in defining the corridor, has been shown in Refs. 94 and 95 to be a realistic value within which human beings can still perform.§ With control at the steeper entrance angles, the maximum range capability of the vehicle is limited. This is because a large amount of vehicle kinetic energy is lost during the initial steep dive into the atmosphere.

For supercircular entries, the range capability from about 1500 to 12,000 miles has been achieved in guidance studies to date. This range capability has been demonstrated in Refs. 32 and 33 using the fast-time prediction method. Studies in Ref. 40 have demonstrated that this capability also can be achieved using control about a single nominal trajectory.

The direct descent entries have been the primary object of the supercircular entry studies to date. There has been some consideration of establishing a circular orbit in the skip-out[61, 96] and the use of multiple pass braking,[91, 97] but there has been little analysis of the closed-loop control to mechanize these schemes.

Guidance from Off-Design Entry Conditions

The possibility of an abort during the various missions poses a stringent requirement upon a guidance system: that it be able to cope with those conditions which are extremely far off the design trajectory.[98] In Ref. 28, the control from off-design entries is considered for guidance about a nominal trajectory. The fast-time prediction guidance method is demonstrated for abort control in Ref. 46. In Ref. 49, pilot control is considered in aborts. These studies have demonstrated that the primary guidance system can be designed to handle those entrance conditions which are markedly off-design.

Concluding Remarks

This survey has outlined several guidance and control methods that have been proposed for atmosphere re-entry. Each method can be designed to realize about the same vehicle capabilities. There are some relative advantages and disadvantages of each system, and these are the following:

1) The use of repetitive fast-time prediction provides a flexible guidance scheme in that range prediction as well as the anticipation of acceleration or heating problems can be obtained from any flight condition. This method provides good display information for pilot decisions. The disadvantage is the on-board computer requirements over the other methods. Approximate closed-form solutions can be used to predict those trajectories which can be described analytically.

2) The guidance about nominal trajectories provides a simple guidance method that can be designed to handle many off-design conditions. Fixed feedback gains can be used in some situations, but varying feedback gains provide better overall capability. This method gives relatively little guidance information for pilot decisions compared to the other methods.

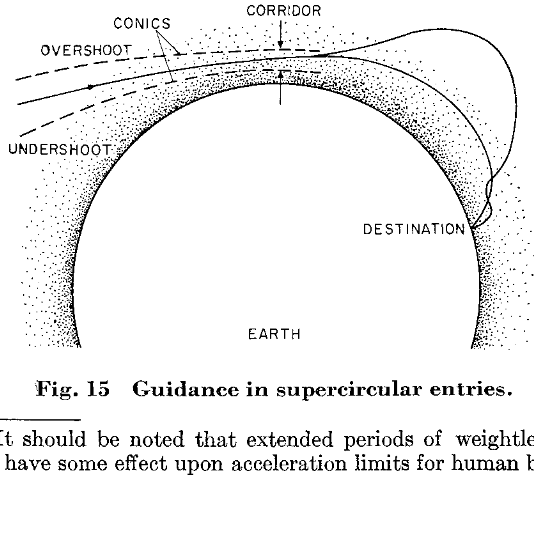


Fig. 15 Guidance in supercircular entries.

§ It should be noted that extended periods of weightlessness may have some effect upon acceleration limits for human beings.

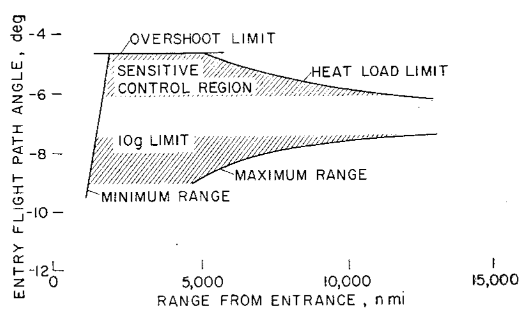


Fig. 16 Typical operational corridor from Ref. 32; u_i = 1.4, h_i = 400,000 ft, L/D = 0.5, W/C_DS = 50 psf.

Independent of the particular guidance method, it has been found important that the four state variables should be represented in the guidance law to insure control in sensitive guidance situations or markedly off-design conditions. Some measurements of the state variables can be made with respect to the local air density as well as to the local position and velocity. Further research is warranted to determine which measurements should be used to represent the state variables and the best use of these variables in the guidance systems.

The more stringent mission profile of the advanced space flight projects also warrants additional research toward the development of new guidance methods and toward the improvement of the systems outlined in this survey.

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